Thermal Conduction as a Wireless Communication Channel

Ruth G. Gebremedhin(ruth.gebremedhin@nyu.edu), Thomas L. Marzetta(tom.marzetta@nyu.edu):–*IEEE Global Communications 2022*

I. Why heat conduction for wireless communication?

- Heat equation new to communication
- Heat conduction is a physical channel
- Covert thermal channel, nanoscale communication
- Space scaling property (cm vs μm)
- Application in intra-chip communication

II. Heat vs Wave Equation

Heat **(Parabolic)**	Wave **(Hyperbolic)**
$\dfrac{\partial T}{\partial t} - \alpha \nabla^2 T = S(\mathbf{x}, t)$	$\dfrac{\partial^2 \phi}{\partial t^2} - c^2 \nabla^2 \phi = S(\mathbf{x}, t)$
→ diffusivity $\alpha[m^2/s]$	→ speed $c[m/s]$

Table: Fundamental properties of the wave and heat Equations

Property	Wave	Heat
(1) Free energy as $t \to \infty$	constant	decreases
(2) Information/Irregularity	transported	lost gradually
(3) Time directional	No	Yes

III. System Block Diagram

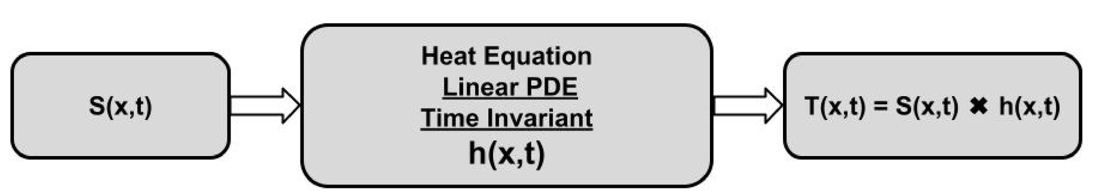

Figure: Linear Systems Model of the Communication System

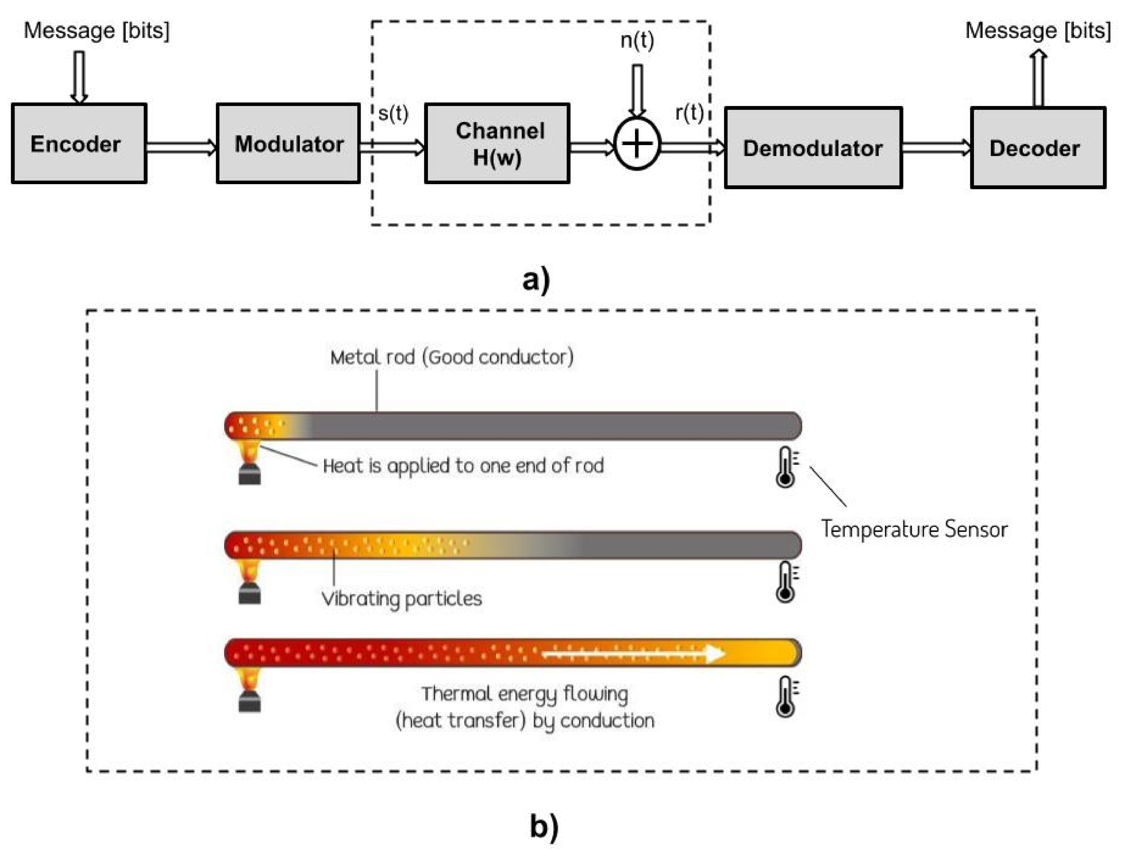

Figure: Block Diagram of the Communication System

IV. Impulse Response

→ $S(\mathbf{x}, t)$, Impulsive heat source at origin
→ $T(\mathbf{x}, t)$, Causal temperature response

$$h(\mathbf{x}, t) = u(t) \frac{e^{\frac{-|\mathbf{x}|^2}{4\alpha t}}}{(4\pi\alpha t)^{\frac{3}{2}}} \quad (1)$$

→ Gaussian in space with standard deviation $\sqrt{2\alpha t}$

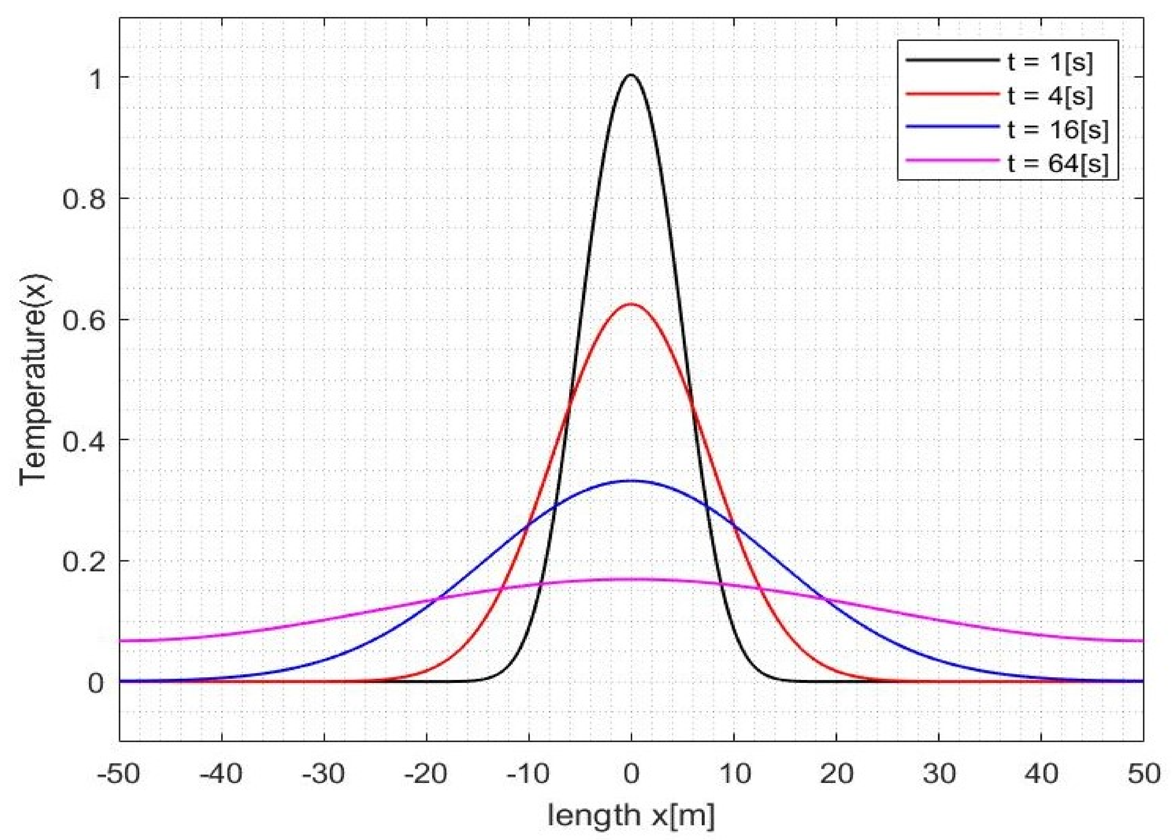

Figure: Impulse Response of the Heat Channel

V. Frequency Response

→ Spherical symmetry in space and wave-number
→ Region of convergence $\Im\{\omega\} = 0$, $\Re\{\omega\} \geq 0$

$$H(x, y, z, \omega) = H(0, 0, R, \omega) = \frac{e^{(i-1)\sqrt{\frac{\omega}{2\alpha}}|R|}}{4\pi\alpha|R|} \quad (2)$$

→ Exponentially decaying function of frequency

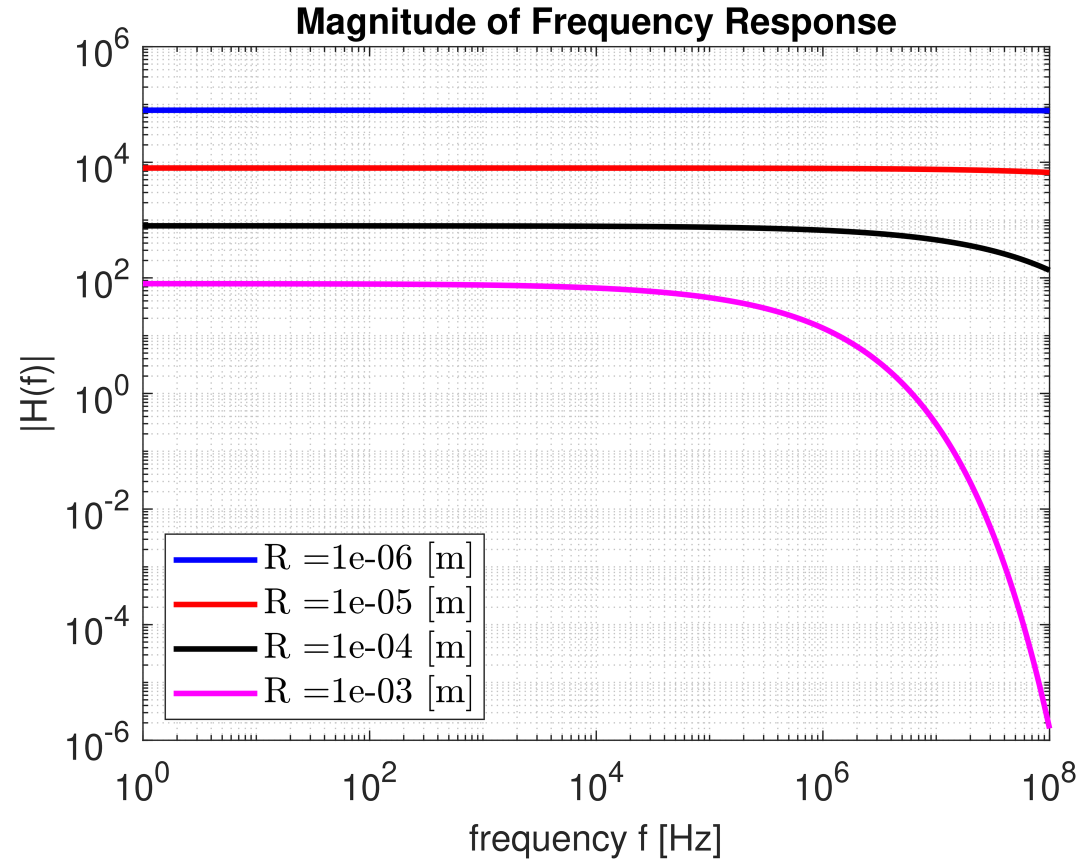

Figure: Magnitude of Frequency Response of the Heat Channel

Main Contribution

The main contribution of this paper is the use of linear system theory to analytically derive the thermal channel's frequency response and capacity from physical principles governing heat conduction.

VI. Analytical Results

Channel Capacity (AWGN)

- Power is absolute value of input, $E|X(t)| \leq P$
- Exact solution for capacity is undetermined
- A lower bound is determined by an optimal Gaussian input, $X_i \sim \mathcal{N}(0, \frac{\pi}{2}P^2)$

Space-Time Scaling

- Heat equation has quadratic scaling in space-time
- Scaling space $|\mathbf{x}|$ by 2 and time t by 4 scales capacity by 4
- Scaling holds though capacity is undetermined

Effective Bandwidth

- Uniquely, total power constrains the bandwidth
- $|H(f)|$, magnitude of the channel's frequency response is monotonically decreasing
- Sub-channels at high frequencies will not be used

VII. Lower Bound of Channel Capacity

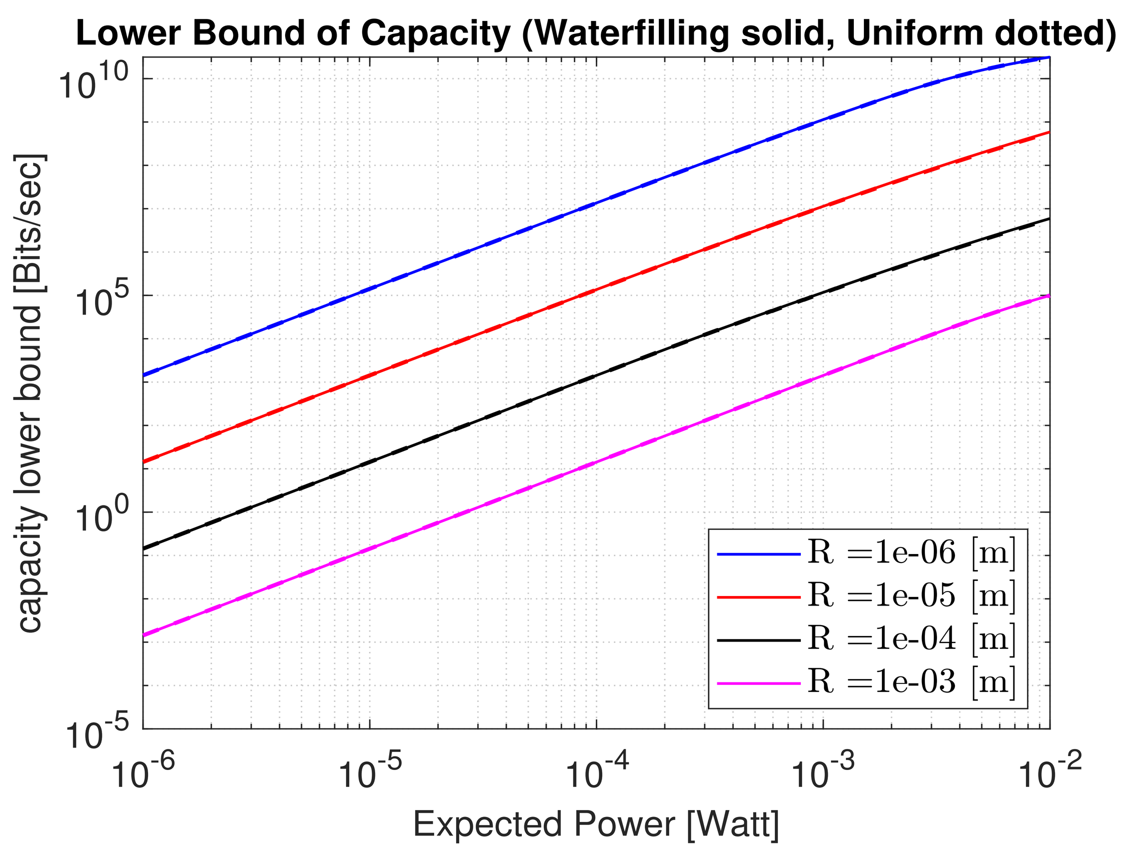

Figure: Lower Bound of the Heat Channel's Capacity

VIII. Effective Bandwidth

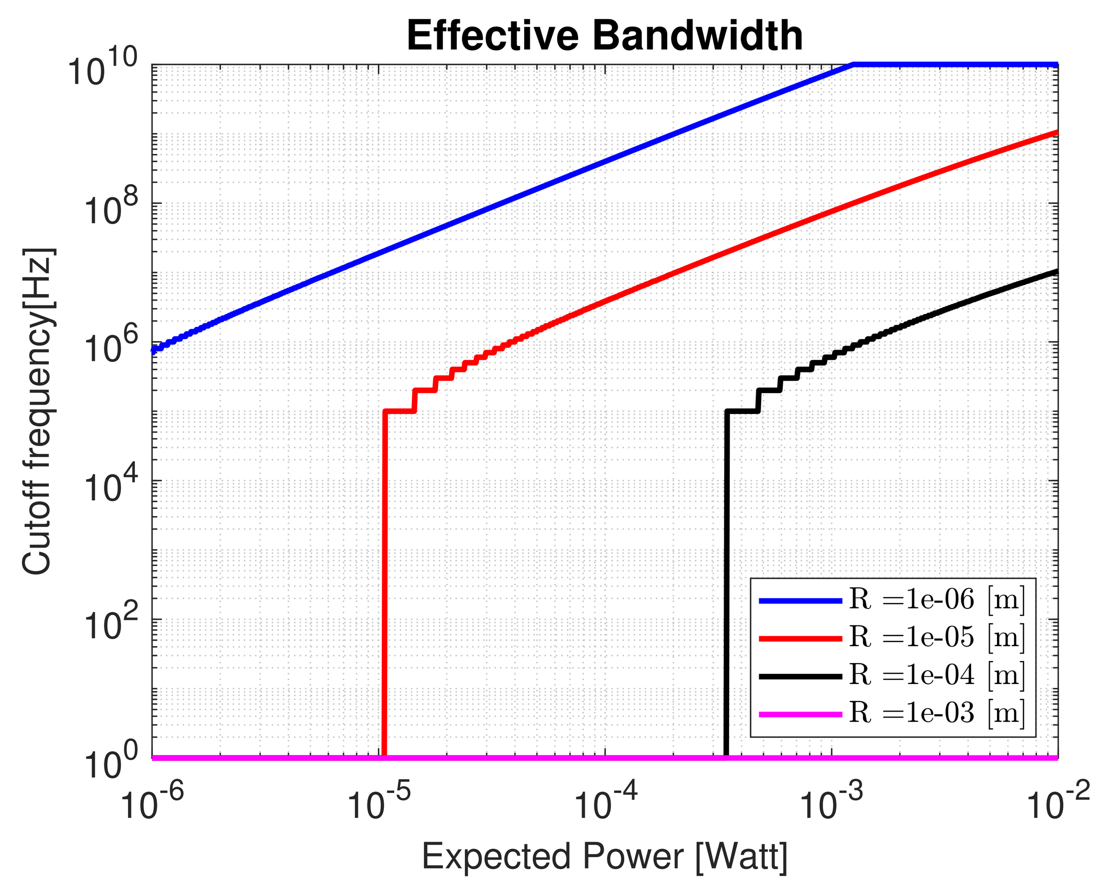

Figure: Effective Bandwidth of the Heat Channel

IX. Conclusion and Discussion

- The thermal channel is fundamentally different from typical wireless channels.
- Parabolic space-time and capacity scaling enables applications in intra-chip communication.
- Bandwidth is less valuable for the thermal channel.
- The thermal channel brings about information theoretic problems that remain to be explored.
- The infinite delay spread of the thermal channel makes OFDM impractical. Hence, a practical modulation/coding scheme needs to be devised.
- The role of the thermal diffusivity α remains to be explored.
- Multiple-input-multiple-output (MIMO) aspects of the thermal channel are yet to be investigated.

References

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