Thermal Conduction as a Wireless Communication Channel

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Outline

Motivation

Why heat conduction for wireless communication?

- Heat equation new to communication but a physical channel
- Nanoscale communication and covert channels
- Space scaling property for capacity (mm vs μm)
- Application in intra-chip communication

Covert Heat Channels and NanoNetworks

Achieved bit Rates

- Guri et al.[3]: two computers ($\approx 0 - 40cm$), 1-8 bits per hour
- Masti et al.[4]: Intel Xeon server multiple cores, 12.5 bps

Capacity

- Zander et al.[6]: intermediate hosts/anonymous servers, 20.5 bits per hour
- Bartoloni et al.[2]: quad-core Intel Core i7-4710MQ, $\approx$300 bits per second (achieved 45bps)

Pierobon et al[5] and Akyildiz et al [1]

- Modulate the diffusion of molecules
- Heat Equation models Diffusion

Problem Definition

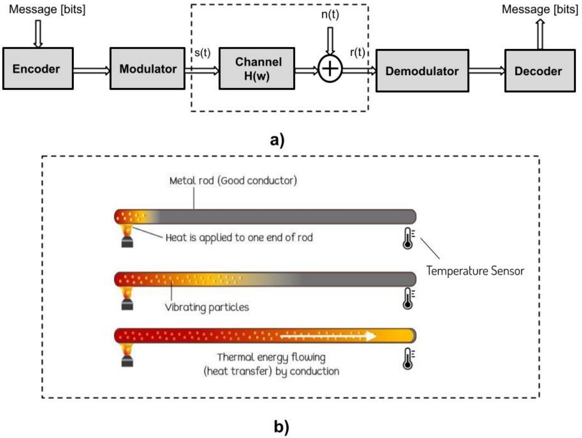


Figure 1: Block Diagram of the Communication System

Contribution

Contribution

The main contribution of this paper is to theoretically derive the channel frequency response and the channel capacity from physical principles governing heat conduction.

Heat vs Wave Equation

Heat Equation (Parabolic):

$$\frac{\partial T}{\partial t} - \frac{\kappa}{c_p \rho}\nabla^2 T = \frac{Q(\mathbf{x}, t)}{c_p \rho} \qquad \frac{\partial T}{\partial t} - \alpha \nabla^2 T = S(\mathbf{x}, t)$$

$Q(\mathbf{x}, t)[J/m^3 s]$ and $S(\mathbf{x}, t)[K/s]$ is the rate of heat generated.

Wave Equation (Hyperbolic):

$$\frac{\partial^2 \phi}{\partial t^2} - c^2 \nabla^2 \phi = S(\mathbf{x}, t)$$

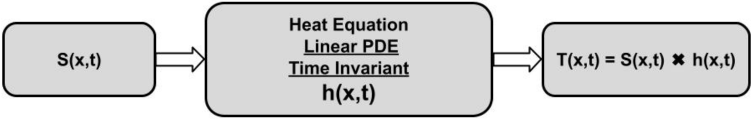
$c\,[m/s]$ is the speed of propagation through the medium.

Heat vs Wave Properties

Property	Wave	Heat
(1) Well-posed for	all time	$t > 0$
(2) Time directional	No	Yes
(3) Free energy as $t \to \infty$	constant	decreases
(4) Information/Irregularity	transported	lost gradually

Table 1: Fundamental properties of the wave and heat Equations

Linear System Approach



→Space-Time Fourier Transform

$$H(k_x,\omega) = \mathscr{F}_t \mathscr{F}_x \{h(x,t)\} \quad and \quad h(x,t) = \mathscr{F}_t^{-1} \mathscr{F}_x^{-1} H(k_x,\omega)$$

Impulse Response

Impulse Response $h(\mathbf{x}, t)$

Source $S(\mathbf{x}, t) = \delta(t)\delta(x)\delta(y)\delta(z)$, Temperature $T(\mathbf{x}, 0) = 0, \forall x$, $T(\pm\infty, t) = 0, \forall t$

1. Forward space Fourier of heat equation
2. PDE $\rightarrow$ ODE, solving ODE gives $H(\mathbf{k}, t)$
3. Inverse space Fourier of $H(\mathbf{k}, t)$

$$h(\mathbf{x}, t) = u(t)\frac{e^{\frac{-|\mathbf{x}|^2}{4\alpha t}}}{(4\pi\alpha t)^{\frac{3}{2}}}$$

$\rightarrow$ Gaussian in space with standard deviation $\sqrt{2\alpha t}$

Impulse Response Figure

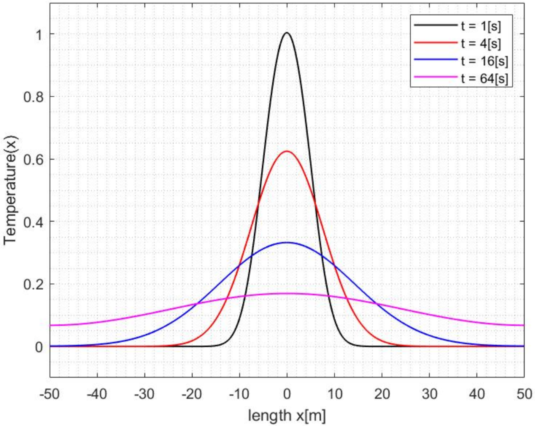


Figure 2: Impulse Response of the Heat Channel

Frequency Response

Frequency Response $H(\mathbf{R}, \omega)$

$\rightarrow$ Forward time Fourier of $H(\mathbf{k}, t)$ gives $H(\mathbf{k}, \omega)$

$\rightarrow$ Spherical symmetry in wave-number implies symmetry in space

$\rightarrow$ Region of convergence $\Im\{\omega\} = 0\,, \Re\{\omega\} \geq 0$

$$H(x, y, z, \omega) = H(0, 0, R, \omega) = \frac{e^{(i-1)\sqrt{\frac{\omega}{2\alpha}}R}}{4\pi\alpha|R|}$$

$$|H(x, y, z, \omega)| = |H(0, 0, R, \omega)| = \frac{e^{-\sqrt{\frac{\omega}{2\alpha}}|R|}}{4\pi\alpha|R|}$$

$\rightarrow$ Magnitude is a monotonically decreasing function of frequency.

Magnitude of Frequency Response

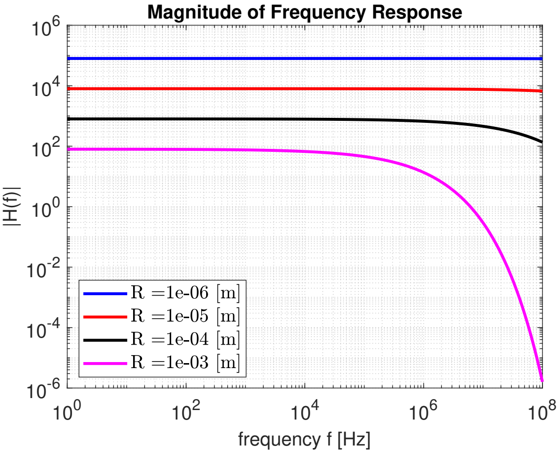


Figure 3: Magnitude of Frequency Response of the Heat Channel

Distributed Source

→ Analytical convolution hard since impulse response is Gaussian

→ Consider $S(\mathbf{x}, t)$ in $|z| \leq R_0$, what is $T(\mathbf{x}, t)$ when $|z| \geq R_0$?

→ Temperature distribution complicated for distances $|z| \leq R_0$.

→ Simple weighted superposition of "pseudo plane waves" $e^{\pm iz\beta}$ when $|z| > R_0$.

$$T(x, y, z, \omega) = \frac{i}{2\alpha} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S' \, \frac{dkx \, dky \, e^{i(k_x x + k_y y)}}{(2\pi)^2 \beta} \tag{1}$$

$$S' = \begin{cases} e^{iz\beta} S(k_x, k_y, [k_z = \beta], \omega) & \text{when } z > R_0 \\ e^{-iz\beta} S(k_x, k_y, [k_z = -\beta], \omega) & \text{when } z < -R_0 \end{cases} \tag{2}$$

→ Temperature field reproduced by two planar sources

Capacity Lower Bound and Scaling

Channel Capacity (AWGN)

- Power is absolute value of input, $E|X(t)| \leq P$
- Exact solution for capacity is undetermined

$$C_{shannon} = \max_{f(x_1,x_2,\ldots x_k):\sum E[X_i^2]\leq P} I(X_1, X_2, \ldots .X_k; Y_1, Y_2, \ldots .Y_k)$$

- A lower bound is determined by an optimal Gaussian input, $X_i \sim \mathcal{N}(0, \frac{\pi}{2}P^2)$

Space-Time and Capacity Scaling

- Heat equation has quadratic scaling in space-time
- Scaling space $|\mathbf{x}|$ by 2 and time t by 4 scales capacity by 4
- Scaling holds though capacity is undetermined

Lower Bound for Capacity

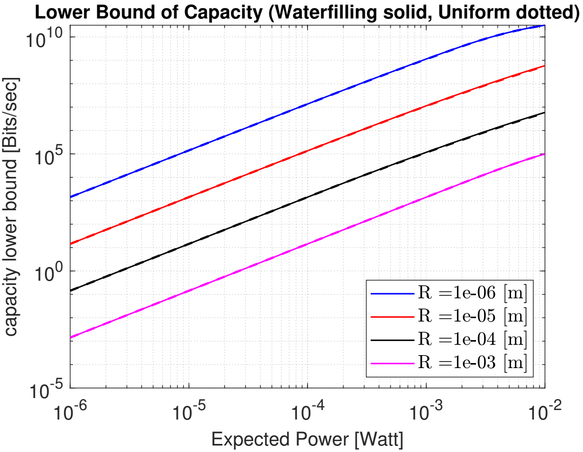
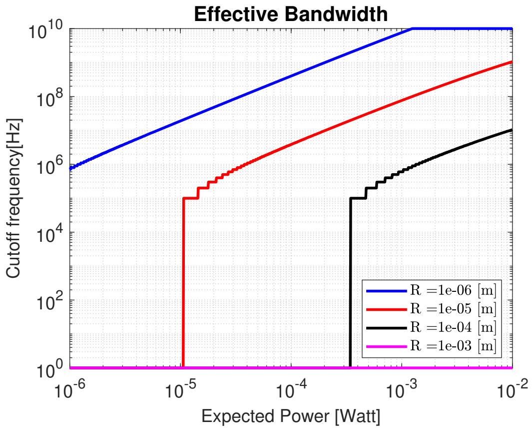


Figure 4: Lower Bound of the Heat Channel's Capacity

Effective Bandwidth

- Uniquely, total power constrains the bandwidth
- Sub-channels at high frequencies will not be used since magnitude is monotonically decreasing



Conclusion

- The thermal channel is fundamentally different from typical wireless channels.
- Parabolic space-time and capacity scaling enables applications in intra-chip communication.
- Bandwidth is less valuable for the thermal channel.
- The thermal channel brings about information theoretic problems that remain to be explored.
- The infinite delay spread of the thermal channel makes OFDM impractical. Hence, a practical modulation/coding scheme needs to be devised.
- The role of the thermal diffusivity α remains to be explored.
- Multiple-input-multiple-output (MIMO) aspects of the thermal channel are yet to be investigated.

Thank You!

[1] Ian F Akyildiz, Josep Miquel Jornet, and Massimiliano Pierobon. "Propagation models for nanocommunication networks". In: *Proceedings of the Fourth European Conference on Antennas and Propagation*. IEEE. 2010, pp. 1–5.

[2] Davide B Bartolini, Philipp Miedl, and Lothar Thiele. "On the capacity of thermal covert channels in multicores". In: *Proceedings of the Eleventh European Conference on Computer Systems*. 2016, pp. 1–16.

[3] Mordechai Guri et al. "Bitwhisper: Covert signaling channel between air-gapped computers using thermal manipulations". In: *2015 IEEE 28th Computer Security Foundations Symposium*. IEEE. 2015, pp. 276–289.

[4] Ramya Jayaram Masti et al. "Thermal covert channels on multi-core platforms". In: *24th {USENIX} Security Symposium ({USENIX} Security 15)*. 2015, pp. 865–880.

[5] Massimiliano Pierobon and Ian F Akyildiz. "A physical end-to-end model for molecular communication in nanonetworks". In: *IEEE Journal on Selected Areas in Communications* 28.4 (2010), pp. 602–611.

[6] Sebastian Zander, Philip Branch, and Grenville Armitage. "Capacity of temperature-based covert channels". In: *IEEE communications letters* 15.1 (2010), pp. 82–84.